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Optimal Power Control Algorithm for Cognitive Radio based on Stacklberg Game

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Abstract—*Cognitive radio networks (CRNs) are applied to solve the problem of spectrum scarcity. Power control is the key technologies in CRNs and efficient power control scheme can decrease the interference to other users and save the power consumption. A critical design challenge for cognitive radio networks is to establish a balance between transmit power and interference. In recent years, several techniques for regulating the transmit power of secondary users in cognitive radio networks have been proposed. The primary user (PU) can admit secondary users (SUs) to access by pricing their interference power under the interference power constraint. The interaction between the PU and the SUs is modeled as a Stacklberg game. The revenue function of the PU is expressed as a convex function of SU's transmitting power by backward induction. Then, we represent the property of optimal transmit power of the SUs, we propose a price-based power control algorithm to maximize the revenue of the BS by using convex optimization. Simulation results show the proposed algorithm improves the revenue of both the BS and SUs.*

Keyword: *Power control, cognitive radio, price, Stackelberg Game*

1. INTRODUCTION

With the growth of wireless applications, the radio spectrum becomes more crowded. It has become a bottleneck to limit the continuous development of wireless mobile communication and services. However, The Federal Communication Commission (FCC) found that spectrum resources of partial licensed frequency have not been fully utilized. So, Cognitive radio (CR) is the key technology, initially proposed by J. Mitola in 1999 to use the underutilized portion of the licensed band opportunistically [1]

CR is a Software Defined Radio (SDR), which change its transmission parameter according to its environment in which it operates. SDR is ''Radio in which some or all of the physical layer function are software defined''. In CRNs, there are two types of users: Primary User (PUs) who has the license of the spectrum and Secondary Users (SUs) who opportunistically utilize the licensed spectrum. There are two different scenarios in spectrum sharing: overlay scenario and underlay scenario [2]. In the overlay spectrum sharing network, the SUs use spectrum sensing to identify the spectrum holes. The SUs can used the licensed spectrum to transmit the data as long as the PUs are inactive. The underlay network, where SUs can use licensed spectrum without causing harmful interference to the PUs, can improve the spectral efficiency and provide more opportunities to SUs.

In traditional wireless communication system, many power control algorithm have been proposed. Transmit power of secondary users (SUs) in cognitive radio networks will inevitably introduce interference to primary users. Hence, a critical design challenge for cognitive radio is to establish a balance between transmits power and interference. The authors [3] proposed a joint spectrum bidding and service pricing model for IEEE 802.22-based cognitive wireless networks. The authors [4] explore the pricing issue for the power control problem in code division multiple access (CDMA) based CRNs. In [5] the optimal investment and pricing decisions in CRNs under spectrum supply uncertainty were addressed.

In this paper, we investigate the price- based power control in CDMA based CRNs. Since the utility of the base station (BS) is non-convex function, it is difficult to find the optimal pricing scheme, and the authors proposed a sub-optimal proportionate pricing algorithm to maximize the revenue of the BS. By characterizing the property of the transmit power of SUs under the optimal pricing scheme, we propose a price-based power control algorithm to find the optimal price for each SU. Simulation results show that the proposed pricing scheme can improve the utilities of both the BS and SUs.

The rest of the paper is organized as follows: In section II System model is presented. In section III, the price- based power control algorithm is proposed. Simulation results are given in section IV. Section V concludes the paper

2. SYSTEM MODEL

We consider an uplink transmission for CDMA based CRNs. The PUs is licensed to transmit to its BS, and n SUs need to pay the BS for their uplink transmissions. The link gain between the SU i and the base station is denoted by h_i(i=1,…… n).L is the spreading gain. The interference power constraint (IPC) of SUs to the BS is T. The PU will charge the i_{th}SU λ_i per unit interference power.

We model the strategy between the BS and SUs as a Stackelberg game [6]. The BS is the leader in this game. It chooses a price for each SU to maximize its own revenue under IPC. The SUs are the followers of the game. After the BS chooses the price for each SU, the SU will decide the transmit power to maximize its utility based on non-cooperative power control game. The problem of the BS is as follows:

Maximize

$$u_p(\lambda_1,\ldots.\lambda_n)=\sum_{i=1}^{n} h_i\lambda_i\, \mathrm{p}_\mathrm{i}^2 \quad (1)$$

subject to

$$\sum_{j=1}^{n} h_j\, p_j = T,\, p_j \geq 0\ j = 0, \ldots \ldots, n \quad (2)$$

Where T is the IPC at the BS, λ_i is the price charged i_{th} SU per unit interference power and p_{max} is the maximal allowed receive power for each SU at the BS. Constraint (2) means that the total interference power made by SUs should be below a given threshold T to ensure the SUs' transmission would not cause unendurable interference to the PUs. Constraint (3) means that the interference power of each SU to the BS should be less than p_{max} to guarantee the fairness among SUs. The utility of the i_{th} SU has two parts: one is the income from the transmit rate achieved at the BS when it transmits at a given power p_i $(i = 1, \ldots., n)$, the other is the payment to the BS.

The SINR of the i_{th} SU at the BS is as follows:

$$\gamma_\mathrm{i} = \frac{\mathrm{Lh_i p_i}}{\sum_{\mathrm{j}\neq\mathrm{i}} \mathrm{h_j p_j} + \sigma^2} \quad (3)$$

Where p_i is the transmit power of the i_{th} SU, P= $(p_1, \ldots \ldots .. p_n)$ is the transmit power of all SUs and σ^2 can be is the interference caused by the PUs and the ambient noise at the BS.

The utility of the i_{th} SU has two parts: one is the income from the transmit rate achieved at the BS where it transmits at a given power p_i, the other is the payment to the BS. Thus, the utility of SU i is given by

$$u_i(p, \lambda_i) = w_i \log\big(1 + \gamma_i(pz)\big) - h_i\lambda_i\, \mathrm{p}_\mathrm{i}^2 \quad (4)$$

Where w_i is the preference factor of the i_{th} SU for the unit rate. The optimization problem for the i_{th} SUs is as follows:

Maximize

$$u_i(p, \lambda_i)$$

Subjected to

$$p_i \geq 0 \quad (5)$$

3. PRICE BASED POWER CONTROL ALGORITHEM

In this section, an optimal price algorithm has purposed for the BS to maximize its revenue according to the property of the transmit power of SUs under the optimal price. The relationship among the transmit power of SUs for the given price of λ_i (i = 1,. . . n) is:

By using the optimal condition for the i_{th} SU in (6), we have

$$\frac{\partial u_i(p_i, p_{-i})}{\partial (p_i)} = \frac{w_i L h_i}{\sum_{j\neq i} h_j p_j + \sigma^2 + L h_i p_i} - 2h_i\lambda_i\, p_i = 0 \quad (6)$$

Then we get the following equation:

$$\left(\sum_{j\neq i} h_j p_j + \sigma^2 + L h_i p_i\right) = \frac{w_i L h_i}{2h_i\lambda_i\, p_i} \quad (7)$$

From equation (6) we get the following equation

$$\frac{w_i L h_i}{2(\sum_{j\neq i} h_j p_j + \sigma^2 + L h_i p_i)} = h_i\lambda_i\, p_i \quad (8)$$

Multiple both the sides by p_i, we have following identities:

$$\frac{w_i L h_i p_i}{2(\sum_{j\neq i} h_j p_j + \sigma^2 + L h_i p_i)} = h_i\lambda_i\, \mathrm{p}_\mathrm{i}^2 \quad (9)$$

It means that the revenue of the BS gets from the i_{th} SU can be expressed as: $w_i L h_i p_i / 2(\sum_{j\neq i} h_j p_j + \sigma^2 + L h_i p_i)$. Then put (9) into (1), the revenue of the BS can be rewritten as follows:

Maximize

$$\sum_{i=1}^{n} \frac{w_i L h_i p_i}{2(\sum_{j\neq i} h_j p_j + \sigma^2 + L h_i p_i)} \quad (10)$$

Subject to

$$\sum_{j=1}^{n} h_j\, p_j = T \quad (11)$$

$$p_j \geq 0, j = 1 \quad (12)$$

We substitute $\sum_{j=1}^{n} h_j\, p_j = T$ into (10). Therefore, the optimal solution to (10) - (12) is equivalent to the following problem

Maximize

$$\sum_{i=1}^{n} \frac{w_i L h_i p_i}{2(T + (L-1) h_i p_i + \sigma^2)}$$

Subjected to

$$\sum_{j=1}^{n} h_j\, p_j = T$$

$$p_j \geq 0, j = 1 \quad (13)$$

It can be verified that the object function of (13) is a concave function. So (13) is a concave maximization problem. Using Karush-Kuhn- Tucker (KKT) conditions, we can derive the solution to (13) as follows:

Theorem 1: Let $(p_1, \ldots \ldots, p_n)$ be the optimal solution to (19), the transmit power of the i_{th} SU is given by

$$p_i = \frac{Max\left\{\frac{\sqrt{\frac{Lw_i(T+\sigma^2)}{2\mu}}-(T+\sigma^2)}{L-1},0\right\}}{h_i} \qquad (14)$$

Where μ is the solution of the following equation:

$$\sum_{i=1}^{n} max\left\{\frac{\sqrt{\frac{Lw_i(T+\sigma^2)}{2\mu}}-(T+\sigma^2)}{L-1},0\right\} = T \qquad (15)$$

Proof: See Appendix A.

From theorem 1, we give a optimal price-based power control algorithm for the BS and the corresponding power allocation for each SU by Algorithm 1. From (14), the optimal power for some SUs with the lower preference factor may be zero. Therefore, the BS might not admit some SUs to maximize its own revenue.

Algorithm*:* optimal price based power control algorithm:

Initialization: set $p_i = Max\left\{\sqrt{\frac{Lw_i(T+\sigma^2)}{2\mu}} - (T+\sigma^2), 0\right\}/h_i$, $(i = 1, \dots\dots, n)$, where μis the solution to (15) which can be solved by using bisection or any other root finding algorithm.

for all $i \in \{1, \dots\dots, n\}$ **do**

$$p_i^* = p_i$$

end for

Output: For $i = 1, \cdots, n$, the optimal price for SU i is given by $\lambda_i^* = w_i L h_i / 2p_i^* (\sum_{j\neq i} h_j p_j^* + \sigma^2 + L h_i p_i^*)$, and the power transmit by SU i is given by p_i^* .

4. SIMULATION RESULTS

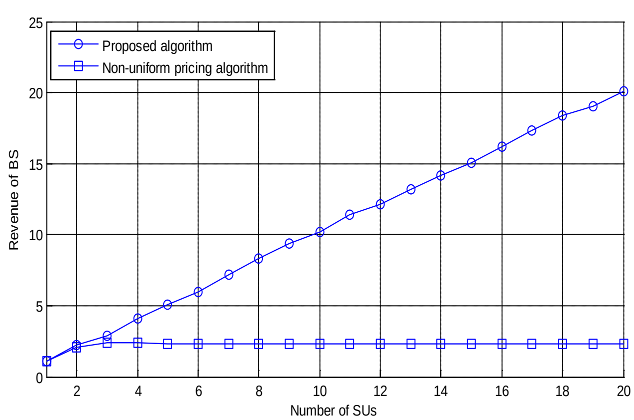

Fig. 1: Revenue of BS versus number of SUs.

In this section, the performance of proposed algorithm is evaluated by comparing it with the non uniform pricing algorithm proposed in [7]. The simulation parameters are the following: $L = 128, \sigma^2 = 10^{-12}W$, the number of SUs is 20, the preference factor for all the SUs is set to be 1. w_i is the uniform distribution in $[0, 300], h$ is uniform distribution in $[0, 1]$. All simulations are averaged by 10^4 realizations. Fig. 1 shows that the revenue of BS verses the number of SUs. The revenue of the BS gained by both the algorithm increase as the number of SUs increases. This is because the BS has more freedom to choose the SUs to maximize its revenues as the number of SUs increases.

The revenue of the BS obtained by the proposed algorithm outperforms the non-uniform pricing algorithm when the number of SUs is larger than one. Fig. 2 shows the sum revenue of SUs versus the number of SUs The sum revenue of SUs decreases for both algorithms when the number of SUs ranges from 1 to 2. When the number of SUs is larger than 2, the sum revenue of SUs increases as the number of SUs increases. This is because the increase rate for sum rata

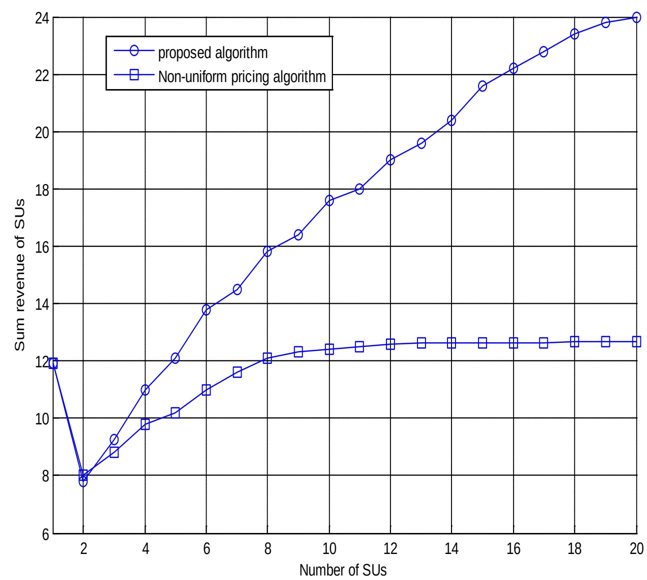

Fig. 2: Sum revenue of SUs versus number of SUs.

of SUs is less than the payoff to the PU when the number of SUs changes from 1 to 2. The proposed algorithm outperforms the non-uniform pricing algorithm in terms of the sum revenue of SUs when the number of SUs is larger than 2. Moreover, the non-uniform pricing algorithm will be saturated, and the proposed pricing algorithm will not be saturated.

5. CONCLUSION

In this paper, we model the price-based power control in CRNs by the Stackelberg game. We characterize the revenue of the PU as a function of the transmit power of the SUs. The optimal condition of maximizing the revenue of the BS is described. Simulation results show that the proposed pricing algorithm improves the revenue of both the BS and SUs.

APPENDIX A

PROOF OF THEOEM 1

Proof: the optimization problem given in the equation (13) is a convex optimization problem. We can express it in the standard form as

Minimize

$$-\sum_{i=1}^{n} \frac{w_i L h_i p_i}{2(T+(L-1)h_i p_i+\sigma^2)} \quad (1)$$

Subjected to

$$\sum_{j=1}^{n} h_j\, p_j = T \quad (2)$$

$$p_j \geq 0, j = 1, \dots \dots, n \quad (3)$$

The Lagrangian can be written as

$$£ = -\sum_{i=1}^{n} \frac{w_i L h_i p_i}{2(T+(L-1)h_i p_i+\sigma^2)} + \mu\left(\sum_{j=1}^{n} h_j\, p_j - T\right) \quad (4)$$

Where μ is a dual variable. In order to derive the dual function we need to minimize the Lagrangian over p_i. The dual function is given by

$$g(\mu) = min_{p\geq 0}\, £(p, \mu)$$

Observe that the constraint $p_i \geq 0$ which is we did not dualize now appears when evaluating the dual function.

The dual function can be written as

$$g(\mu) = -\mu T - \sum_{i=1}^{n} min_{p\geq 0} \left\{\frac{w_i L h_i p_i}{2(T+(L-1)h_i p_i+\sigma^2)} - \mu h_i p_i\right\} \quad (5)$$

It is easy to minimize the second term since except for $p_i \geq 0$, it is an unconstrained problem. There are two cases: either $p_i = 0$, or $p_i > 0$, that must hold that

$$\frac{d}{dp_i}\left(\frac{w_i L h_i p_i}{2(T+(L-1)h_i p_i+\sigma^2)} - \mu h_i p_i\right) = 0$$

After solving the above equation, we get

$$p_i = \frac{\sqrt{\frac{Lw_i(T+\sigma^2)}{2\mu}} - (T+\sigma^2)}{(L-1)h_i}$$

Since p_i cannot be less than 0, the optimum value of p_i as a function of μ is

$$p_i = \frac{Max\left\{\frac{\sqrt{\frac{Lw_i(T+\sigma^2)}{2\mu}} - (T+\sigma^2)}{L-1}, 0\right\}}{h_i} \quad (6)$$

More interestingly however, we can directly solve for μby using the expression for p_i into the (2). By combining (2) and (6), we get the following equation

$$\sum_{i=1}^{n} max\left\{\frac{\sqrt{\frac{Lw_i(T+\sigma^2)}{2\mu}} - (T+\sigma^2)}{L-1}, 0\right\} = T \quad (7)$$

This can be solved for μ very easily using bisection or any root finding algorithm.

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